Extensive-Form Games

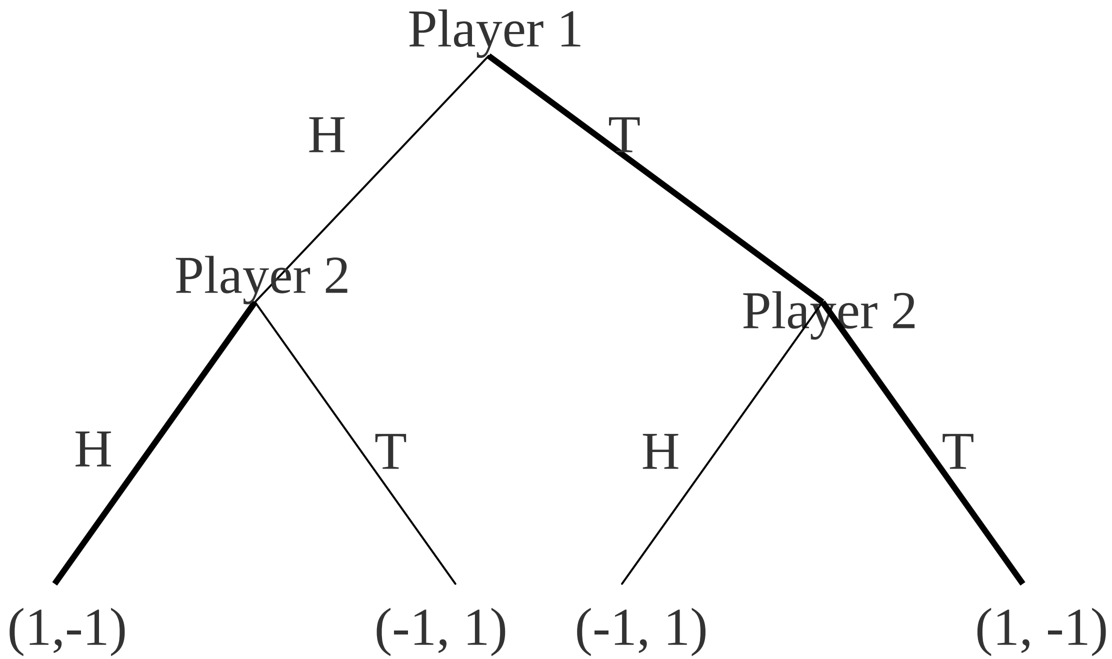
- An Extensive-Form Game consist of the following elements:
  - A set of players *N*
  - A set of histories *H* (all possible sequences of moves)
  - A player function *P*, which assigns a player (decision-maker) to every history
  - A payoff function, which assigns payoffs for each player to every **terminal** node
- It differs from a Normal-Form Game
  - It is dynamic (players move in some order)
  - Players may observe histories (what happened so far in the game)
  - Every time a player makes a move, that move can be conditioned on the history

Game Trees

- A game tree is a graph that represents an extensive-form game, like a game matrix for normal-form games
- In practice, this representation is used only for relatively simple games
- Game Trees consist of:
  - Nodes (Decision Nodes, Terminal Nodes), that represent histories
  - Branches (Arcs), that represent the possible decisions (moves, actions) at a decision node

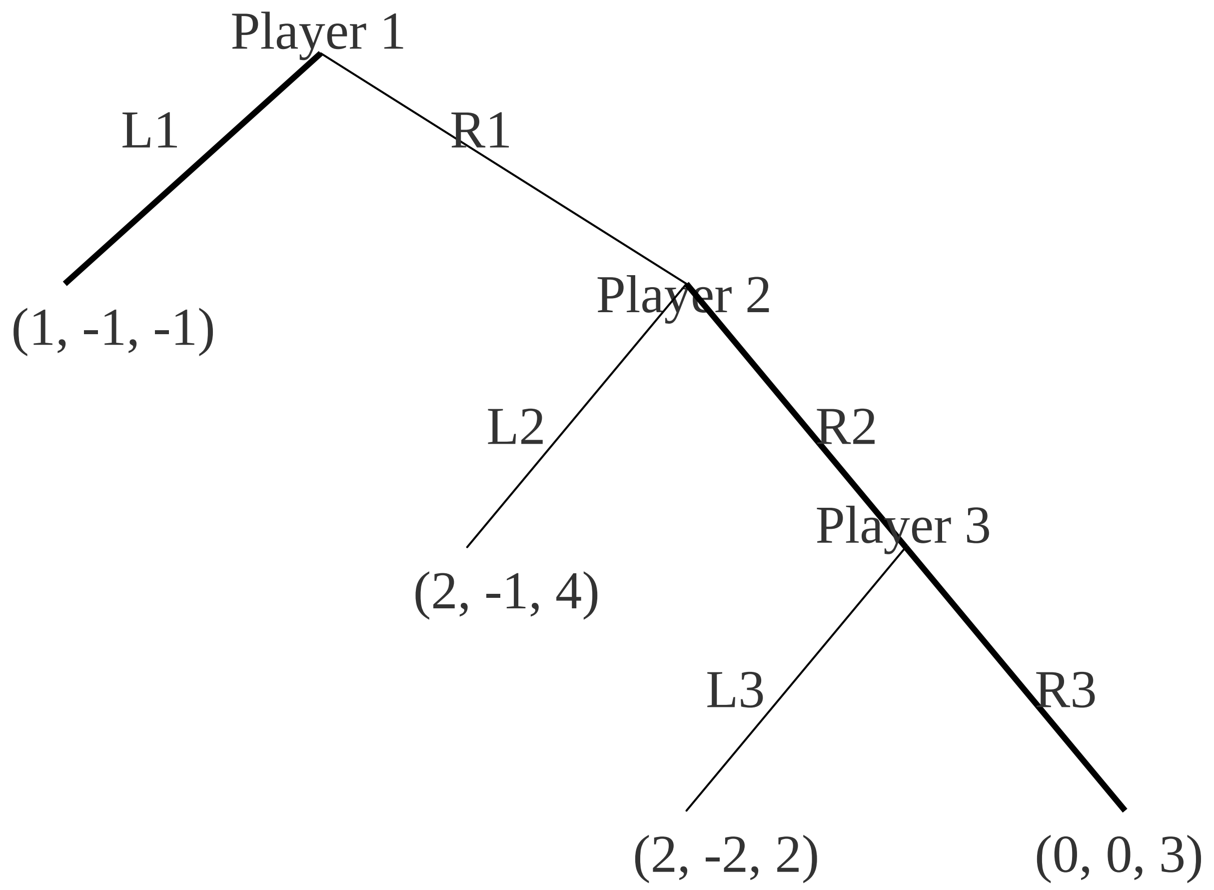
Game Trees - Examples

- Biased matching pennies



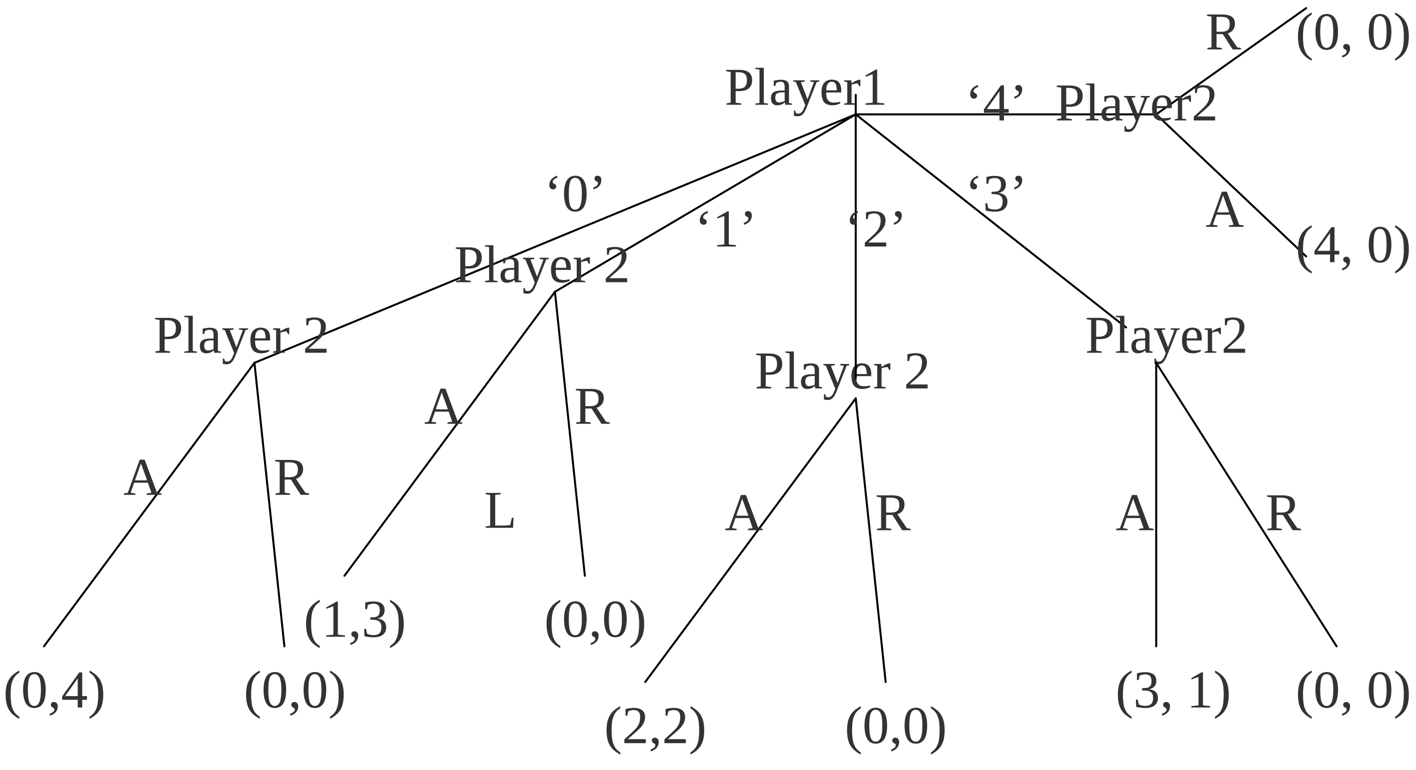
Game Trees - Examples

- A 3-player game



Game Trees - Examples

- Ultimatum game

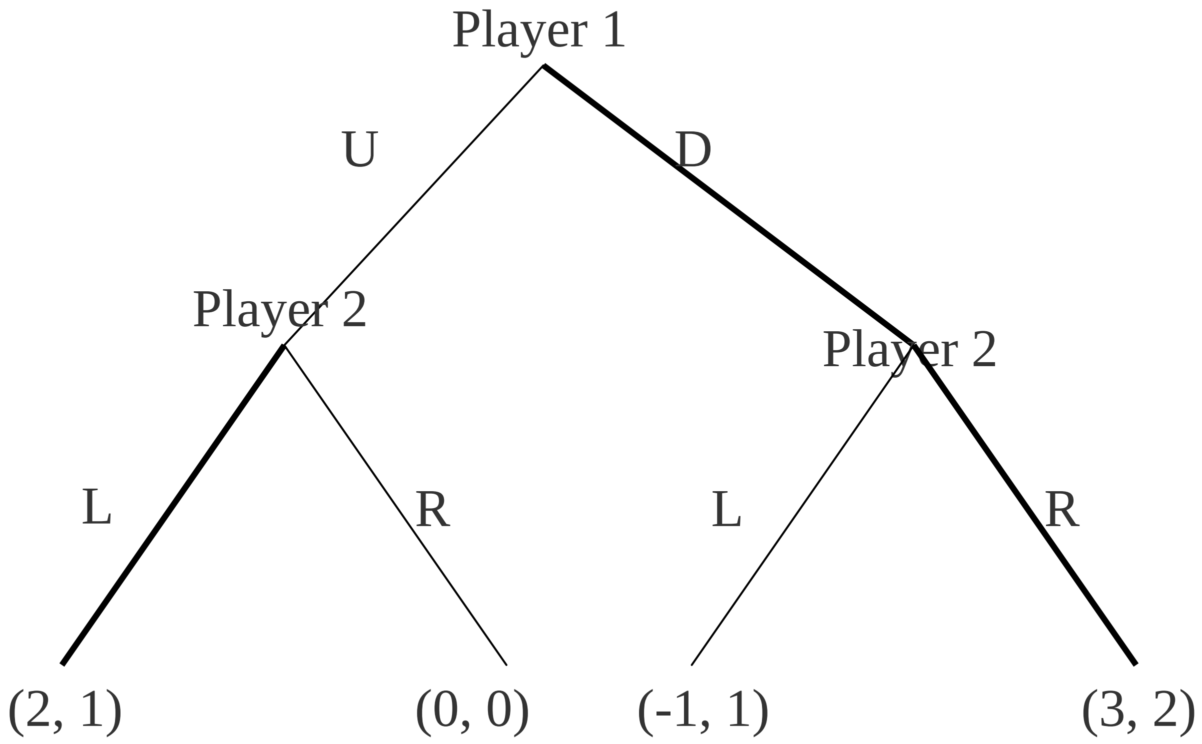


Strategies in ext.-form games

- In extensive-form games, a (pure) strategy is a complete game plan, i.e. it assigns a (pure) decision to every possible decision node
- In the 3-player game, each player has only two pure strategies
- In the biased matching pennies, player 1 has 2 strategies, player 2 has 4
- In the ultimatum game, player 1 has 5, player 2 has 32 strategies

Reducing to Normal Form

- The following game reduces to...



Reducing to Normal Form

- Something like this:

		Player 2			
Player 1		LL	LR	RL	RR
	U	$\underline{2}, \underline{1}$	$2, \underline{1}$	$\underline{0}, 0$	$0, 0$
	D	$-1, 1$	$\underline{3}, \underline{2}$	$-1, 1$	$\underline{3}, \underline{2}$

Subgame Perfect Nash Equilibrium (SPNE)

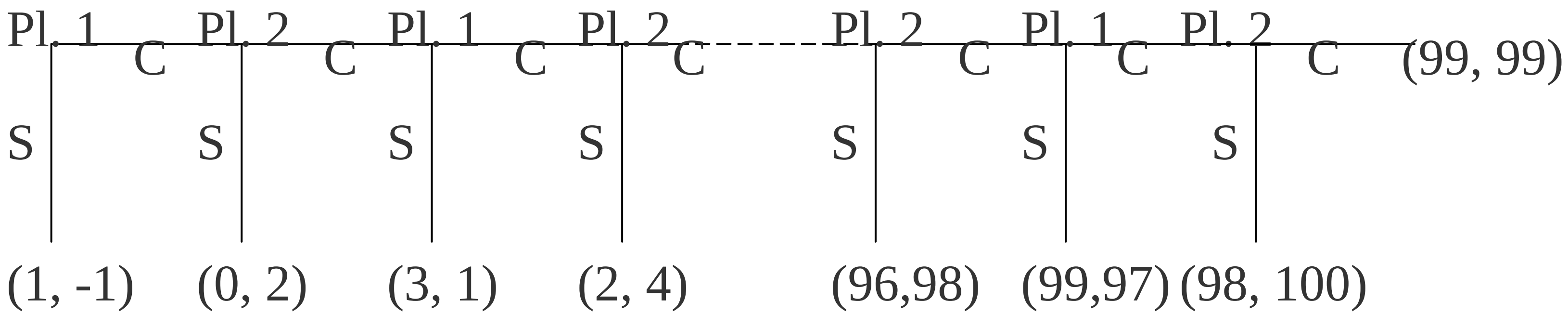
- A **subgame** of an extensive-form game (with perfect information) is a game which begins at any non-terminal history and contains all nodes (histories) and possible moves that can follow after that history.
- A **subgame-perfect** Nash Equilibrium (SPNE) is pair of strategies (pure or mixed) which forms a NE in every subgame. SPNE is a refinement of NE.
- The optimal algorithm of identifying SPNE is **backward induction.** You start from finding best-responses in the smallest (final) subgames and then consider ever bigger subgames, fixing the best-responses which have been identified in smaller subgames. Problem: cannot be used in infinite-horizon games.

Finding SPNE

- In the last example, only (D,LR) is a SPNE, even though there are 3 NE
- In the 3-player game (L1, R2, R3) is a SPNE, but (R1,L2,L3) is a NE that is not subgame perfect
- In the biased matching pennies game, in all SPNEs player 2 plays TH (player 1 is indifferent between T and H)

The drawbacks of SPNE

- Find the SPNE of the Centipede Game



- SPNE={SS..S,SS..S}